Analysis of Adaptive Data-Reusing Normalised Least Mean Square Switching Kronecker Ll Filters

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Abstract: This paper introduces an adaptive data-reusing switching Kronecker Ll filtering framework based on data-reusing method which can switch between sub-filters tuned by means of data-reusing normalised least mean square (DR-NLMS-SKF) algorithm. Data-reusing approach is applied by mean of number of reused tap-weight update per sample in order to support the robust characteristics and smoothing filtering. The coefficients of proposed DR-NLMS-SKF algorithm are the samples of bounded real-valued function. The proposed DR-NLMS-SKF filter can be designed in form of a stochastic gradient filter. Simulation results show that the proposed filter can obtain the robustness, smoothing and sharpening filtering method in some applications.

Index Terms: Switching filter, Kronecker Ll filter, Data-reusing method, Normalised least mean square algorithm, Adaptive algorithm.

I. Introduction

Applications of Image processing have been widely used in many area of signal processing [1]. Based on order filters, nonlinear filterings have been useful in many applications [2]. According to linear filters, these are able to design for tuning specific property of frequency such as low-pass, band-pass and high-pass properties, even it cannot remove totally the impulse noise.

In order to achieve the good convergence, the data-reusing (DR) mechanism is based on *a posteriori* error adaptation that presents the better convergence than standard *a priori* error [3]. Thus,the desired response and input vector are used in order to refine the estimate filtering [4]. The idea of data-reusing least mean square (DR-LMS) algorithm has been presented that the reuse of each received symbol based on DR-LMS algorithm allows faster convergence than least mean square (LMS) algorithm when compared on the requirement of training sequences [5].

Consequently, the switching method of two sub-Ll filters has been presented in [6]. This is suitable for both edge preserving characteristics or noise smoothing filters by tuning the value of K parameter for each sub-Ll filters with the method of mean squre error criterion. In [7], an adaptive DR-LMS based on switching Kronecker Ll filters has been presented in terms of the smoothing and sharpening.

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In this paper, the objective is to derive the adaptive switching Kronecker Ll filter based on data-reusing method based on the normalised least mean square (DR-NLMS) algorithm as a flexible image processing filter with the properties of smoothing, edge preserving and robust characteristics. This paper is organised as follows. Section II describes about the switching Kronecker Ll filters and Section III explains about data-reusing algorithm based on normalised least mean square (DR-NLMS) algorithm. The adaptive DR-NLMS algorithm based on switching Kronecker Ll filters are proposed. Simulation results and conclusion are detailed in Section V and Section VI, respectively.

II. Switching Kronecker Ll filters

The coefficients of Kronecker Ll filters are defined by the product of α_i and β_j, where these coefficients of α_i and β_j referring to the position in the window of input signal and the order in the local window, respectively.

The output of Kronecker Ll filter $y_{i,j}(n)$ is given as [8]

$$y_{i,j}(n) = \sum_{s=1}^{m} \alpha_i \beta_j x_{i,j}(n) \ , \tag{1}$$

where $x_{i,j}(n)$ occupies the ranked sample.

The idea is that how to switch the sub-filters output using a signal activity information, so called *a local information* as presented in [6].

So, the switching Kronecker Ll filter output $\hat{y}_{i,j}(n)$ can be expressed as

$$\hat{y}_{i,j}(n) = \mathcal{K}_{T,S}(i,j)\tilde{y}_{p_{i,j}}(n) + \big(1 - \mathcal{K}_{T,S}(i,j)\big)\tilde{y}_{s_{i,j}}(n) \ , \tag{2}$$

where $\tilde{y}_{p_{i,j}}(n)$ and $\tilde{y}_{s_{i,j}}(n)$ are the outputs of Kronecker Ll filters referring to a smooth and preserving signal as

$$\tilde{y}_{p_{i,j}}(n) = \sum_{j=1}^{m} \alpha_i \, \beta_j \, x_{i,j}(n) \ , \qquad (3)$$

where the product of α_i and β_i are the Kronecker Ll filter coefficients for preserving and

$$\tilde{y}_{s_{i,j}}(n) = \sum_{j=1}^{m} \tilde{\alpha}_i \, \tilde{\beta}_j \, x_{i,j}(n) \ , \qquad (4)$$

where the product of $\tilde{\alpha}_i$ and $\tilde{\beta}_i$ are the Kronecker Ll filter coefficients for smooth signal.

A local information $\mathcal{K}_{T,S}(i,j)$ is defined by [9]

$$\mathcal{K}_{T,S}(i,j) = \frac{\sigma^2_{T,S}(i,j)}{\sigma^2_{T,S}(i,j) + \sigma^2_{\eta}} \ , \qquad (5)$$

where S and T denote the higher rank and the lower rank. The estimated local variance $\sigma^2_{T,S}(i,j)$ of original signal within the window defined as

$$\sigma^2_{T,S}(i,j) = \max\{\text{Var}_{T,S}(i,j) - \sigma^2_{\eta}\} \ , \qquad (6)$$

where σ^2_{η} is the variance of additive Gaussian noise and $\text{Var}_{T,S}(i,j)$ is the local variance within the window.

III. Data-Reusing Normalised Least Mean Square Algorithm

In [3], data-reusing (DR) algorithms effectively operate between the symbol instants n and $n+1$ by combining the recursive mode of learning based on the *a priori* output error and iterative mode of learning based on the *a posteriori* errors.

Following [5], the update weight vector $\mathbf{w}_{t+1}(n)$ in the data-reusing normalised least mean square (DR-NLMS) algorithm is introduced as

$$\mathbf{w}_{t+1}(n) = \mathbf{w}_t(n) + \mu \, \frac{\mathbf{x}(n) \; e_t(n)}{\mathbf{x}(n)^2} \ , \qquad (7)$$

$$e_t(n) = d(n) - \mathbf{x}^T(n) \; \mathbf{w}_t(n) \ , \qquad (8)$$

where $\mathbf{w}_1(n) = \mathbf{w}(n)$, $\mathbf{w}_{L+1}(n) = \mathbf{w}(n+1)$, $t = 1, \ldots, L$ and t denotes as the order of data-reuse iteration. The parameter μ is a step-size. The vector $\mathbf{x}(n)$ is the input signal vector and $d(n)$ is the desired signal. The error $e_t(n)$ is the t^{th} data-reusing estimated error.

For the *a priori* and *a posteriori* mode of operaiton, the relationship at $t = 2$ is as

$$\begin{aligned} e_2(n) &= d(n) - \mathbf{x}^T(n)\mathbf{w}_2(n) \\ &= d(n) - \mathbf{x}^T(n)[\mathbf{w}_1(n) + \mu\mathbf{x}(n)e_1(n)] \\ &= e_1(n)\,[1 - \mu\,\mathbf{x}^T(n)\mathbf{x}(n)] \ , \end{aligned} \qquad (9)$$

and the t^{th} data-reusing error can be given as

$$e_t(n) = e(n)\,[1 - \mu\,\mathbf{x}^T(n)\mathbf{x}(n)]^{t-1} \ , \ t = 1, \ldots, L \ . \qquad (10)$$

Then, the final estimated error $\sum_{t=1}^{L} e_t(n)$ with L data-reusing iterations is defined as [4]

$$\begin{aligned} \sum_{t=1}^{L} e_t(n) &= \sum_{t=1}^{L} e(n) \; [1 - \mu \, \mathbf{x}^T(n)\mathbf{x}(n)]^{t-1} \\ &= \frac{e(n)\big[1 - \big(1 - \mu \, \mathbf{x}^T(n)\mathbf{x}(n)\big)^L\big]}{\mu \, \mathbf{x}^T(n)\mathbf{x}(n)} \ . \end{aligned} \qquad (11)$$

Therefore, the tap-weight DR-NLMS vector $\mathbf{w}(n+1)$ can be recursively expressed as

$$\begin{aligned} \mathbf{w}_{L+1}(n) &= \mathbf{w}_L(n) + \mu \frac{e_L(n) \; \mathbf{x}(n)}{\mathbf{x}(n)^2} \\ &= \mathbf{w}_{L-1}(n) + \mu \, \frac{(e_{L-1}(n) + e_L(n)) \; \mathbf{x}(n)}{\mathbf{x}(n)^2} \\ &= \mathbf{w}(n) + \mu \, \sum_{t=1}^{L} \frac{e_t(n) \; \mathbf{x}(n)}{\mathbf{x}(n)^2} \\ &= \mathbf{w}(n+1) \ , \end{aligned} \qquad (12)$$

where $\sum_{t=1}^{L} e_t(n)$ is given in (11).

IV. Adaptive Data-Reusing Normalised Least Mean Square Switching Kronecker Ll Filters

Following [11], this section introduces an adaptive switching Kronecker Ll filter based on data-reusing normalised least mean square (DR-NLMS) algorithm which is given on sample-by-sample basis as

$$\begin{aligned} \hat{y}_{i,j}(n) = K \; &\alpha_i(n)\beta_j(n)x_{i,j}(n) \\ &+ (1-K) \; \tilde{\alpha}_i(n)\tilde{\beta}_j(n)x_{i,j}(n) \, , \end{aligned} \qquad (13)$$

where K is a robust information of $\mathcal{K}_{T,S}(i,j)$, $0 < K < 1$. The parameters T and S are defined as the lower rank and higher rank of signals, respectively.

The estimated error $\xi(n)$ using the robust information K in (5) is given as

$$\begin{aligned} \xi(n) = d(n) - K \, &\sum_{n=1}^{N} \alpha_i(n)\beta_j(n)x_{i,j}(n) \\ &- (1-K) \, \sum_{n=1}^{N} \tilde{\alpha}_i(n)\tilde{\beta}_j(n)x_{i,j}(n) \ . \end{aligned} \qquad (14)$$

By means of mean square error (MSE) criterion, the objective function to be minimised is given as

$$J(n) = \frac{1}{2} \sum_{n=1}^{N} \{\, \xi(n) \,\}^2 \ . \qquad (15)$$

Based on DR-NLMS algorithm, the coefficient $\alpha_{t,i}(n)$ can be defined adaptively as [3]

$$\alpha_{t+1,i}(n+1) = \alpha_{t,i}(n) - \frac{\mu}{x^2_{i,j}(n)} \, \nabla_{\alpha_i(n)} J(n) \ , \qquad (16)$$

where $t = 1, \ldots, L$ and $\alpha_{1,i}(n) = \alpha_i(n)$. The cost function $J(n)$ is defined in (15).

The gradient $\nabla_{\alpha_i(n)} J(n)$ is given by differentiating the squared estimation error $\xi^2_{\alpha_i,t}(n)$ as

$$\xi_{\alpha_i,t}(n) = d(n) - K \sum_{n=1}^{N} \alpha_{t,i}(n)\; \beta_j(n)\; x_{i,j}(n) - (1-K) \sum_{n=1}^{N} \tilde{\alpha}_i(n)\tilde{\beta}_j(n) x_{i,j}(n) \;, \quad (17)$$

with respect to $\alpha_{t,i}(n)$. This yields

$$\nabla_{\alpha_i(n)} J(n) = -K\; \beta_j(n)\; x_{i,j}(n)\; \xi_{\alpha_i,t}(n) \;, \quad (18)$$

where μ is the step-size parameter. The parameter K is given in (5).

Consequently, the DR-NLMS switching Kronecker Ll filter for $\alpha_{t,i}(n)$ can be performed recursively as

$$\alpha_{t+1,i}(n+1) = \alpha_{t,i}(n) + \frac{\mu}{x^2_{i,j}(n)} K\beta_j(n) x_{i,j}(n) \xi_{\alpha_i,t}(n) \;. \quad (19)$$

For preliminary insight, the estimated error $\xi_{\alpha_{i,t}}(n)$ at symbol t is presented for the $t = 2$ case.

The relationship between the *a priori* and *a posteriori* errors at $t = 2$ for $\alpha_{t,i}(n)$ is given as

$$\begin{aligned}
\xi_{\alpha_{i,2}}(n) &= d(n) - K \sum_{n=1}^{N} \alpha_{2,i}(n)\; \beta_j(n) x_{i,j}(n) \\
&\quad - (1-K) \sum_{n=1}^{N} \tilde{\alpha}_i(n)\tilde{\beta}_j(n) x_{i,j}(n) \\
&= d(n) - K \sum_{n=1}^{N} \big[\alpha_{1,i}(n) \\
&\quad + \mu K\, \beta_j(n) x_{i,j}(n) \xi_{\alpha_i,1}(n)\big] \beta_j(n) x_{i,j}(n) \\
&\quad - (1-K) \sum_{n=1}^{N} \tilde{\alpha}_i(n)\tilde{\beta}_j(n) x_{i,j}(n) \\
&= \xi_{\alpha_{i,1}}(n)\; \left[1 - \mu K \beta_j(n) x^2_{i,j}(n)\right] \;. \quad (20)
\end{aligned}$$

Therefore, the relationship $\xi_{\alpha_i,t}(n)$ at t for the coefficients $\alpha_i(n)$ can be expressed as

$$\xi_{\alpha_i,t}(n) = \xi(n)[1 - \mu K \beta_j(n) x^2_{i,j}]^{t-1} \;, \quad (21)$$

where $\xi(n)$ is given in (14).

The relationship between $\alpha_{L+1,i}(n+1)$ and $\alpha_i(n+1)$ is given by

$$\begin{aligned}
\alpha_{L+1,i}(n+1) &= \alpha_{L,i}(n) + \frac{\mu}{x^2_{i,j}(n)}\; K\; \beta_j(n)\; \xi_{\alpha_i,L}(n)\; x_{i,j}(n) \\
&= \alpha_{L-1,i}(n) + \frac{\mu\; K\; \beta_j(n)}{x^2_{i,j}(n)} \left(\xi_{\alpha_i,L-1}(n) + \xi_{\alpha_i,L}(n)\right) x_{i,j}(n) \\
&= \alpha_i(n) + \frac{\mu\; K\; \beta_j(n)}{x^2_{i,j}(n)} \sum_{t=1}^{L} \xi_{\alpha_i,t}(n)\; x_{i,j}(n) \\
&= \alpha_i(n+1) \;, \quad (22)
\end{aligned}$$

where $\xi_{\alpha_i,t}(n)$ is given in (21).

The update coefficient $\alpha_i(n+1)$ of DR-NLMS switching Kronecker Ll filter can be defined by

$$\alpha_i(n+1) = \alpha_i(n) + \frac{\mu\; K\beta_j(n)}{x_{i,j}(n)} \sum_{t=1}^{L} \xi_{\alpha_i,t}(n)\; x_{i,j}(n) \,. \quad (23)$$

The sum of L data-reusing estimated error $\sum_{t=1}^{L} \xi_{\alpha_i,t}(n)$ for $\alpha_i(n)$ is defined as

$$\begin{aligned}
\sum_{t=1}^{L} \xi_{\alpha_i,t}(n) &= \sum_{t=1}^{L} \xi(n)\; [1 - \mu\; K\beta_j(n) x^2_{i,j}]^{t-1} \\
&= \frac{\xi(n)\big[1 - \big(1 - \mu\; K\beta_j(n) x^2_{i,j}\big)^L\big]}{\mu\; K\beta_j(n) x^2_{i,j}} \,. \quad (24)
\end{aligned}$$

In a similar fashion, the coefficient $\tilde{\alpha}_i(n+1)$ of DR-NLMS switching Kronecker Ll filter can be expressed as

$$\tilde{\alpha}_i(n+1) = \tilde{\alpha}_i(n) + \frac{\mu K\tilde{\beta}_j(n)}{x^2_{i,j}(n)} \sum_{t=1}^{L} \xi_{\tilde{\alpha}_i,t}(n)\, x_{i,j}(n) \;, \quad (25)$$

where the sum of L data-reusing estimated error $\sum_{t=1}^{L} \xi_{\alpha_i,t}(n)$ for $\tilde{\alpha}_i(n)$ can be expressed as

$$\begin{aligned}
\sum_{t=1}^{L} \xi_{\tilde{\alpha}_i,t}(n) &= \sum_{t=1}^{L} \xi(n)\, [1 - \mu\; K\tilde{\beta}_j(n) x^2_{i,j}]^{t-1} \\
&= \frac{\xi(n)\big[1 - \big(1 - \mu\; K\; \tilde{\beta}_j(n) x^2_{i,j}\big)^L\big]}{\mu\; K\tilde{\beta}_j(n) x^2_{i,j}} \;. \quad (26)
\end{aligned}$$

Therefore, the coefficient $\beta_{t,i}(n)$ can be defined adaptively as [3]

$$\beta_{t+1,i}(n+1) = \beta_{t,i}(n) - \mu\, \nabla_{\beta_j(n)} J(n) \;, \quad (27)$$

where $t = 1, \ldots, L$ and $\beta_{1,i}(n) = \beta_i(n)$.

The update coefficient $\beta_j(n+1)$ of DR-NLMS switching Kronecker Ll filter can be defined by

$$\beta_j(n+1) = \beta_j(n) + \frac{\mu K\alpha_i(n)}{x^2_{i,j}(n)} \sum_{t=1}^{L} \xi_{\beta_{j,t}}(n) x_{i,j}(n) \;, \quad (28)$$

where the sum of L data-reusing estimated error $\sum_{t=1}^{L} \xi_{\beta_{j,t}}(n)$ for $\tilde{\beta}_j(n)$ can be expressed as

$$\begin{aligned}
\sum_{t=1}^{L} \xi_{\beta_{j,t}}(n) &= \sum_{t=1}^{L} \xi(n)\; [1 - \mu\; K\; \alpha_i(n)\; x^2_{i,j}]^{t-1} \\
&= \frac{\xi(n)\big[1 - \big(1 - \mu\; K\alpha_i(n)\; x^2_{i,j}\big)^L\big]}{\mu\; K\alpha_i(n) x^2_{i,j}} \,. \quad (29)
\end{aligned}$$

The update coefficient $\tilde{\beta}_j(n+1)$ of DR-NLMS switching Kronecker Ll filter can be defined by

$$\tilde{\beta}_j(n+1) = \tilde{\beta}_j(n) + \frac{\mu\; K\; \tilde{\alpha}_i(n)}{x^2_{i,j}(n)} \sum_{t=1}^{L} \xi_{\tilde{\beta}_j,t}(n)\; x_{i,j}(n) \quad (30)$$

and the sum of L data-reusing estimated error $\sum_{t=1}^{L} \xi_{\tilde{\beta}_{j,t}}(n)$ for $\beta_j(n)$ can be expressed as

$$\sum_{t=1}^{L} \xi_{\tilde{\beta}_j,t}(n) = \sum_{t=1}^{L} \xi(n)\ [1 - \mu\ K\ \tilde{\alpha}_i(n)x_{i,j}^2]^{t-1}$$
$$= \frac{\xi(n)\big[1 - \big(1 - \mu\ K\ \tilde{\alpha}_i(n)x_{i,j}^2\big)^L\big]}{\mu\ K\ \tilde{\alpha}_i(n)x_{i,j}^2}\ . \qquad (31)$$

The summary of proposed adaptive data-reusing normalised least mean square algorithm based on switching Kronecker Ll filering (DR-NLMS-SKF) is shown in Table I.

V. Simulation Results

In this section, we simulate the test signal using 8-bit gray-scale of "Peppers" image in order to assess the performance of proposed DR-NLMS switching Kronecker Ll filter that we have discussed. This image is corrupted by multiplicative noise, also known as *speckle noise* that is common beside additive noise with the variance $(\sigma_\eta^2 = 0.06)$ [12]. The different window sizes $(M \times M)$ are of $\{(3\times 3), (4\times 4), (5\times 5)\}$. The initial parameters of filters based on the conventional LMS order statistic (LMS-OS) and LMS Kronecker Ll filters are as $\mu = 0.25$, $L = 3, 4, 5$ and $\alpha(0) = \beta(0) = \tilde{\alpha}(0) = \tilde{\beta}(0) = 1/\sqrt{M}$. Summary of LMS-OS and LMS Kronecker Ll filters are detailed briefly in Appendix.

The criteria have been used in quantitative comparison of proposed filters as follows [13].

1) Mean square error (MSE):

$$\text{MSE} = \frac{1}{N}\sum_{n=1}^{N}\big(\ y(n) - \hat{y}(n)\ \big)^2\ . \qquad (32)$$

2) Root mean square error (RMSE):

$$\text{RMSE} = \sqrt{\frac{1}{N}\sum_{n=1}^{N}\Big(\ y(n) - \hat{y}(n)\ \Big)^2}\ . \qquad (33)$$

3) Signal to noise ratio (SNR) in dB:

$$\text{SNR} = 10\log_{10}\ \frac{\sum_{n=1}^{N}(\ y^2(n)\ )}{\sum_{n=1}^{N}\Big(\ y(n) - \hat{y}(n)\ \Big)^2}\ . \qquad (34)$$

4) Improvement in signal-to-noise ratio (ISNR) in dB:

$$\text{ISNR} = 20\log_{10}\left(\left|\frac{\sigma_\eta}{\text{RMSE}}\right|\right) \qquad (35)$$
$$= 20\log_{10}\left(\left|\frac{\sigma_\eta}{\sqrt{\frac{1}{N}\sum_{n=1}^{N}\Big(\ y(n) - \hat{y}(n)\ \Big)^2}}\right|\right)\ .$$

TABLE I: Summary of proposed adaptive data-reusing normalised least mean square algorithm based on switching Kronecker Ll filering (DR-NLMS-SKF)

For $n = 1, 2, \ldots,$
For $j = 1, 2, \ldots,\ M$
For $i = 1, 2, \ldots,\ N$

1) The output of adaptive DR-NLMS switching Kronecker Ll filter:

$$\hat{y}_{i,j}(n) = K\ \alpha_i(n)\beta_j(n)x_{i,j}(n) + (1-K)\ \tilde{\alpha}_i(n)\tilde{\beta}_j(n)x_{i,j}(n)\ .$$

2) The estimated error using the robust information K:

$$\xi(n) = d(n) - K\sum_{n=1}^{N}\alpha_i(n)\beta_j(n)x_{i,j}(n) - (1-K)\sum_{n=1}^{N}\tilde{\alpha}_i(n)\tilde{\beta}_j(n)x_{i,j}(n)\ .$$

3) The update coefficient $\alpha_i(n)$ of DR-NLMS switching Kronecker Ll filter :

$$\alpha_i(n+1) = \alpha_i(n) + \frac{\mu\ K\beta_j(n)}{x_{i,j}(n)}\sum_{t=1}^{L}\xi_{\alpha_i,t}(n)\ x_{i,j}(n)$$
$$\sum_{t=1}^{L}\xi_{\alpha_i,t}(n) = \frac{\xi(n)\big[1 - \big(1 - \mu\ K\beta_j(n)x_{i,j}^2\big)^L\big]}{\mu\ K\beta_j(n)x_{i,j}^2}\ .$$

4) The update coefficient $\tilde{\alpha}_i(n)$ of DR-NLMS switching Kronecker Ll filter :

$$\tilde{\alpha}_i(n+1) = \tilde{\alpha}_i(n) + \frac{\mu K\tilde{\beta}_j(n)}{x_{i,j}^2(n)}\sum_{t=1}^{L}\xi_{\tilde{\alpha}_i,t}(n)\,x_{i,j}(n)\ ,$$
$$\sum_{t=1}^{L}\xi_{\tilde{\alpha}_i,t}(n) = \sum_{t=1}^{L}\xi(n)\ [1 - \mu\ K\tilde{\beta}_j(n)x_{i,j}^2]^{t-1}$$
$$= \frac{\xi(n)\big[1 - \big(1 - \mu\ K\ \tilde{\beta}_j(n)x_{i,j}^2\big)^L\big]}{\mu\ K\tilde{\beta}_j(n)x_{i,j}^2}\ .$$

5) The update coefficient $\beta_j(n)$ of DR-NLMS switching Kronecker Ll filter :

$$\beta_j(n+1) = \beta_j(n) + \frac{\mu K\alpha_i(n)}{x_{i,j}^2(n)}\sum_{t=1}^{L}\xi_{\beta_j,t}(n)x_{i,j}(n)\ .$$
$$\sum_{t=1}^{L}\xi_{\beta_j,t}(n) = \sum_{t=1}^{L}\xi(n)\ [1 - \mu\ K\ \alpha_i(n)\ x_{i,j}^2]^{t-1}$$
$$= \frac{\xi(n)\big[1 - \big(1 - \mu\ K\alpha_i(n)\ x_{i,j}^2\big)^L\big]}{\mu\ K\alpha_i(n)x_{i,j}^2}\ .$$

6) The update coefficient $\tilde{\beta}_j(n)$ of DR-NLMS switching Kronecker Ll filter :

$$\tilde{\beta}_j(n+1) = \tilde{\beta}_j(n) + \frac{\mu\ K\ \tilde{\alpha}_i(n)}{x_{i,j}^2(n)}\sum_{t=1}^{L}\xi_{\tilde{\beta}_j,t}(n)\ x_{i,j}(n)\ ,$$
$$\sum_{t=1}^{L}\xi_{\tilde{\beta}_j,t}(n) = \sum_{t=1}^{L}\xi(n)\ [1 - \mu\ K\ \tilde{\alpha}_i(n)x_{i,j}^2]^{t-1}$$
$$= \frac{\xi(n)\big[1 - \big(1 - \mu\ K\ \tilde{\alpha}_i(n)x_{i,j}^2\big)^L\big]}{\mu\ K\ \tilde{\alpha}_i(n)x_{i,j}^2}\ .$$

End
End
End

TABLE II: Accuracy of the estimation of proposed data-reusing normalised least mean square switching Kronecker Ll filter (DR-NLMS-SKF) with various L compared with least mean square order statistics (LMS-OS) and least mean square Kronecker Ll at window size of 3×3 and $\sigma_\eta^2 = 0.06$.

Filter	$M \times M$	μ	L	MSE	RMSE	SNR (dB)	ISNR(dB)	PSNR(dB)
LMS-OS	3×3	0.25	-	0.0126	0.1123	13.2253	5.4934	18.2437
LMS Kronecker Ll	3×3	0.25	-	0.0151	0.1227	12.4529	6.2155	17.7847
DR-NLMS Switching Kronecker Ll (DR-NLMS-SKF)	3×3	0.25	3	0.0044	0.0665	17.7786	11.3288	22.7093
		0.25	4	0.0042	0.0646	18.0243	11.5744	22.8574
		0.25	5	0.0040	0.0633	18.1980	11.7482	22.9443

TABLE III: Accuracy of the estimation of proposed data-reusing normalised least mean square switching Kronecker Ll filter (DR-NLMS-SKF) with various L compared with least mean square order statistics (LMS-OS) and least mean square Kronecker Ll at window size of 4×4 and $\sigma_\eta^2 = 0.06$.

Filter	$M \times M$	μ	L	MSE	RMSE	SNR (dB)	ISNR(dB)	PSNR(dB)
LMS-OS	4×4	0.25	-	0.0020	0.0452	21.1300	14.6802	25.5134
LMS Kronecker Ll	4×4	0.25	-	0.0024	0.0488	20.4573	14.1784	24.6129
DR-NLMS Switching Kronecker Ll (DR-NLMS-SKF)	4×4	0.25	3	0.0045	0.0674	17.6599	11.2100	22.6752
		0.25	4	0.0034	0.0581	18.9405	12.4906	23.3155
		0.25	5	0.0027	0.0524	19.8423	13.3924	23.7664

TABLE IV: Accuracy of the estimation of proposed data-reusing normalised least mean square switching Kronecker Ll filter (DR-NLMS-SKF) with various L compared with least mean square order statistics (LMS-OS) and least mean square Kronecker Ll at window size of 5×5 and $\sigma_\eta^2 = 0.06$.

Filter	$M \times M$	μ	L	MSE	RMSE	SNR (dB)	ISNR(dB)	PSNR(dB)
LMS-OS	5×5	0.25	-	0.0045	0.0671	17.6905	11.3880	21.9407
LMS Kronecker Ll	5×5	0.25	-	0.0033	0.0572	19.0853	12.6306	23.3267
DR-NLMS Switching Kronecker Ll (DR-NLMS-SKF)	5×5	0.25	3	0.0062	0.0785	16.3288	9.8789	22.0097
		0.25	4	0.0040	0.0636	18.1606	11.7107	22.9256
		0.25	5	0.0029	0.0534	19.6797	13.2298	23.6851

5) Peak signal-to-noise-ratio (PSNR) in dB:

$$\text{PSNR} = 20\log_{10}\left(\frac{y_{\max}(n)}{\text{RMSE}}\right) \tag{36}$$
$$= 20\log_{10}\left(\frac{y_{\max}(n)}{\sqrt{\frac{1}{N}\sum_{n=1}^{N}\Big(y(n)-\hat{y}(n)\Big)^2}}\right).$$

Tables II - IV provide an improvement of MSE, RMSE and SNR which can be obtained by using the proposed DRNLMS switching Kronecker Ll filters compared with the LMS-based algorithm for Kronecker Ll filters and the summary of the ISNR, PSNR achieved by the adaptive proposed filters using $\mu = 0.25$ at window size of $(3 \times 3), (4 \times 4), (5 \times 5)$ and $L = 3, 4, 5$ are presented, respectively.

It is seen that the proposed DR-NLMS-SKF from Table II can achieve the high SNR, ISNR, PSNR at low window size 3×3 and high L. In Table IV, the SNR of proposed DR-NLMS-SKF can obtain the high SNR, ISNR and PSNR at high window size 5×5.

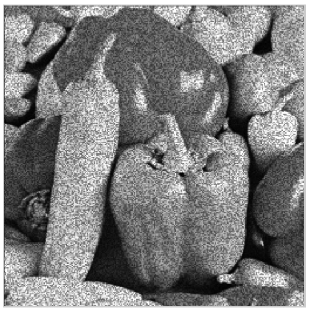
Fig. 1 shows the results of proposed DR-NLMS-SKF algorithm in suppressing noise in “Peppers” image. The original gray-scale image corrupted by multiplicative noise with an uniformly distributed random noise are shown in Fig. 1a, where $\sigma_\eta^2 = 0.06$, respectively.

With the step-size $\mu = 0.25$ in Fig. 1, the proposed filtered images for the window size of 3×3 of LMS order-statistics (LMS-OS) filters and LMS Kronecker Ll are illustrated in Fig. 1b and Fig. 1c, respectively. The filtered images using proposed filters for the window size of 3×3 and $L = 3, 4, 5$ are presented in Fig. 1d, Fig. 1e and Fig. 1f, respectively. According to these results, the measures achieved by the proposed filters give an improvement in terms of signal-to-noise ratio improvement.

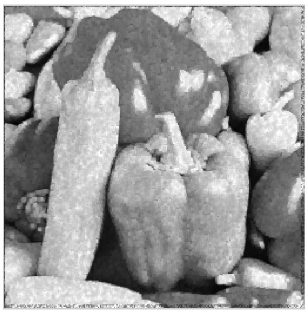
VI. Conclusion

We have derived the adaptive switching Kronecker Ll filters based on the data-reusing normalised least mean square (DR-NLMS-SKF) algorithm whose coefficients are samples of a bounded real-valued function with the properties of robustness by means of mean square error criterion. The data-reusing error has been introduced with the switching Kronecker Ll filters referring to a smooth and preserving data by using a local information.

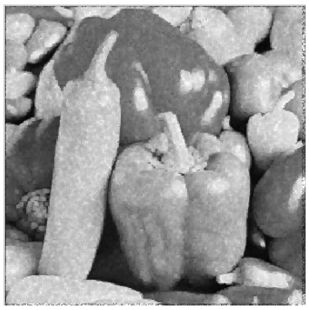
The proposed DR-NLMS switching Kronecker Ll filters can achieve the improvement in terms of signal to noise ration (SNR), improvement of SNR (ISNR) and peak SNR (PSNR) compared with the Kronecker Ll filters based on conventional LMS algorithm.



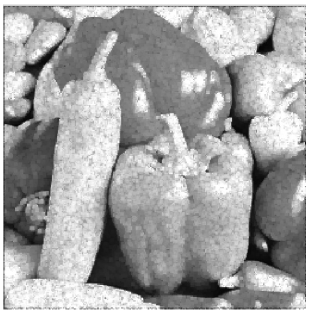
(a) Corrupted "Peppers" image, where $\sigma_\eta^2 = 0.06$.



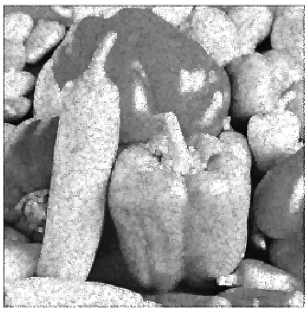
(b) Filtered image using 3×3 LMS order-statistic (LMS-OS) filter



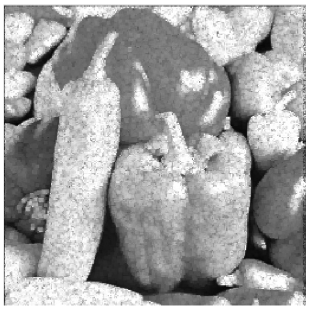
(c) Filtered image using 3×3 LMS Kronecker Ll filter



(d) Filtered image using $L = 3$ and 3×3 DR-NLMS switching Kronecker Ll filter (DR-NLMS-SKF)



(e) Filtered image using $L = 3$ and 3×3 DR-NLMS switching Kronecker Ll filter (DR-NLMS-SKF)



(f) Filtered image using $L = 3$ and 3×3 DR-NLMS switching Kronecker Ll filter (DR-NLMS-SKF)

Fig. 1: Illustration of corrupted and filtered example "Peppers" images

Appendix

A. *LMS Order Statistic (LMS-OS) filter*

Following [6] and [10] , the coefficient $a_i(n)$ of order statistic filter based on the least mean square (LMS-OS) algorithm is denoted as

$$a_i(n+1) = a_i(n) + \mu\, x_{i,j}(n)\, \epsilon(n) \ , \qquad \text{(A.1)}$$

where μ is the step-size and $\epsilon(n)$ is the estimation error on the sample-by-sample basis defined by

$$\epsilon(n) = d(n) - a_i(n)\, x_{i,j}(n) \ . \qquad \text{(A.2)}$$

B. *LMS Kronecker Ll filter*

The coefficient $a_i(n)$ of Kronecker Ll filter based on LMS algorithm is denoted as

$$\alpha_i(n+1) = \alpha_i(n) + \mu\, \beta_j(n)\, x_{i,j}(n)\, \epsilon(n) \ , \qquad \text{(A.3)}$$

$$\beta_i(n+1) = \beta_i(n) + \mu\, \alpha_i(n)\, x_{i,j}(n)\, \epsilon(n) \ , \qquad \text{(A.4)}$$

where $\epsilon(n)$ is the estimation error on the sample-by-sample basis defined by

$$\epsilon(n) = d(n) - \sum_{n=1}^{N} \alpha_i(n)\, \beta_j\, x_{i,j}(n) \ . \qquad \text{(A.5)}$$

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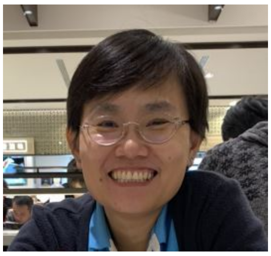

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